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Locally and Colocally Factorable Banach Spaces

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Abstract

We generalize the concept of locality (resp. colocally) to the concept of locally factorable (resp. colocally factorable) such that Theorem 2 is still valid for the new concepts. In addition we show that locally factorable and colocally factorable Banach spaces are inherited by complemented subspace, then we present some examples and establish relations between locally factorable and colocally factorable Banach spaces.

1. Introduction

An operator $T: X \to Y$ of Banach spaces is an **isomorphism** if it is an invertible bounded linear map, T is an into **isometry** if $\|Tx\| = \|x\|$ for every $x \in X$, it is a λ-**isomorphism**, $\lambda > 1$, if T is an isomorphism and $\|T\| < \lambda$, $\|T^{-1}\| < \lambda$.

The distance between two homogeneous maps T_1 and T_2 acting between the same spaces is given by

$$\operatorname{dist}(T_1, T_2) = \sup\left\{\|T_1x - T_2x\| : \|x\| \le 1\right\}.$$

We note that bounded maps are those maps at a finite distance from the zero map, also it should be kept in mind that linear maps are not assumed to be bounded. Let E be a family of finite dimensional Banach spaces, a Banach space X is said to **contain** E **uniformly complemented** if there exists a constant c such that for every $E \in \mathrm{E}$, there is a complemented subspace A of X which is c-isomorphic to E. It is clear that X contains E uniformly complemented if and only if its second dual X^{**} does. A Banach space X is said to be λ-**locally** E if there exists a constant $\lambda > 1$ such that every finite dimensional subspace A of X is contained in a finite dimensional subspace B of X such that $d_{BM}(B, E) < \lambda$, for some $E \in \mathrm{E}$, where $d_{BM}(B, E)$ is the **Banach-Mazur distance** between B and E, and is defined by

$$d_{BM}(B, E) = \inf\left\{\|T\|\|T^{-1}\| : T : B \to E \text{ is an isomorphism of } B \text{ onto } E\right\}.$$

If $\mathrm{E} = \left\{\ell_p^n\right\}_{n=1}^{\infty}$, then X is an L_p**-space**.

A closed subspace Y of a Banach space X is said to be **locally complemented** in X if for every finite dimensional subspace $E \subset X$ there exists an operator $P: E \to Y$ such that P is the identity on $Y \cap E$ with $\|P\| \le M$ for some M independent of E.

A Banach space X is called λ-**colocally** E (or **colocally** E) if there exists a constant $\lambda > 1$ such that every finite dimensional quotient A of X is a quotient of another finite dimensional quotient B of X satisfying $d_{BM}(B, E) < \lambda$ for some $E \in \mathrm{E}$.

The space $L_p(\mu)$, for any measure μ, is both locally and colocally $\left\{\ell_p^n\right\}_{n=1}^{\infty}$.

Let X be a Banach space, C_X be the set of all finite dimensional subspaces A of X directed by the inclusion, and let $\ell_\infty(A; C_X)$ be the collection of all $(X_A)_{A\in C_X} \in \prod_{A\in C_X} A$ such that $\left(\|x_A\|\right)_{A\in C_X}$ is bounded, with norm given by

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$$\left\|(x_A)_{A\in C_X}\right\|_\infty = \sup_{A\in C_X}\|x_A\|.$$

Let U be an ultrafilter on C_X that refines the corresponding order filter, and let $\left(\prod_{A\in C_X} A\right)_U$ be the ultraproduct of the family C_X with respect to the ultrafilter U, that is $\left(\prod_{A\in C_X} A\right)_U$ is the quotient space $\ell_\infty(A;C_X)/N_U$, where

$$N_U = \left\{(x_A)_{A\in C_X} \in \ell_\infty(A;C_X) : \lim_U \|x_A\| = 0\right\}.$$

The elements of $\left(\prod_{A\in C_X} A\right)_U$ are denoted by $(x_A)_U$ and its norm is given by $\left\|(x_A)_U\right\| = \lim_U \|x_A\|$. The map $J_X : X \to \left(\prod_{A\in C_X} A\right)_U$ defined by $J_X(x) = (x_A)_U$, where $x_A = x$, if $x \in A$ and $x_A = 0$, otherwise, is an isometry of X onto a subspace of $\left(\prod_{A\in C_X} A\right)_U$. Moreover the bidual of X is isometrically isomorphic to a quotient of an ultraproduct of the finite dimensional space of X.

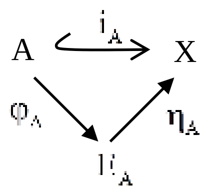
A diagram $0 \longrightarrow Y \xrightarrow{\ i\ } X \xrightarrow{\ q\ } Z \longrightarrow 0$ of quasi Banach spaces and bounded linear operators is called **an exact sequence** if the kernel of each arrow coincides with the image of the preceding one. The open mapping theorem implies that X contains i(Y) and the quotient X/i(Y) is isomorphic to Z. In this case, we shall say that X is a **twisted sum** of Y and Z. Two exact sequences $0 \longrightarrow Y \longrightarrow X_1 \longrightarrow Z \longrightarrow 0$ and $0 \longrightarrow Y \longrightarrow X_2 \longrightarrow Z \longrightarrow 0$ are said to be **equivalent** if there is a bounded linear operator T making the diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & Y & \longrightarrow & X_1 & \longrightarrow & Z & \longrightarrow & 0 \\ & & \| & & T\downarrow & & \| & & \\ 0 & \longrightarrow & Y & \longrightarrow & X_2 & \longrightarrow & Z & \longrightarrow & 0 \end{array}$$

commutative. The open mapping theorem implies that T must be an isomorphism. An exact sequence $0 \longrightarrow Y \longrightarrow X \longrightarrow Z \longrightarrow 0$ is said to **split** if it is equivalent to the trivial exact sequence $0 \longrightarrow Y \longrightarrow Y \oplus Z \longrightarrow Z \longrightarrow 0$, in this case, we say that X is **trivial**. We denote by $\mathrm{Ext}(Z,Y)$ the space of all equivalence classes of locally convex twisted sums of Y and Z. Thus $\mathrm{Ext}(Z,Y) = 0$ means that all locally convex twisted sums of Y and Z are equivalent to the direct sum $Y \oplus Z$.

2. Locally Factorable and Colocally Factorable

Let E be a family of finite dimensional Banach spaces. A Banach space X is said to be **λ-locally E-factorable** (or simply locally E-factorable) if there is a constant $\lambda > 1$ such that for every finite dimensional subspace A of X, there is $E_A \in E$, called **a companion of** A, and there are bounded linear maps $\varphi_A : A \to E_A$, $\eta_A : E_A \to X$ with $\|\varphi_A\| \le \lambda$ and $\|\eta_A\| \le \lambda$ such that $\eta_A \circ \varphi_A = i_A$, where $i_A : A \to X$ is the inclusion map. The maps φ_A, η_A are **the bounded linear factorization** of i_A through E_A, and the diagram



is called **a locally factorable diagram** for A with respect to E_A. Note that a companion $E_A \in E$ of A is not unique. It is clear that if a Banach space X is λ-locally E, it is λ-locally E-factorable. Also, it is obvious that if a Banach space X is locally E-factorable, so is every complemented subspace of X and every Banach space isomorphic to X. On the other hand, a complemented subspace of a Banach space which is locally E need not be locally E also. Indeed, the space $L_p(0,1)$, $1 < p < \infty$, $p \neq 2$, is locally $\{\ell_p^n\}_{n=1}^{\infty}$ and contains a complement subspace A_p which is not locally $\{\ell_p^n\}_{n=1}^{\infty}$ since it is isomorphic to a Hilbert space.

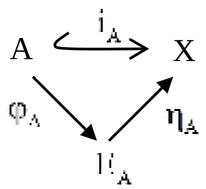
Throughout this paper E and F denote families of finite dimensional Banach spaces and E^* denotes the family of the duals of the spaces in E. We say that E is c-**chained to** F, $c > 1$, if for each $E \in E$, there is $G \in F$ and bounded linear maps $T : E \to G$, $Q : G \to E$ with $\|T\| \leq c$, $\|Q\| \leq c$ such that $Q \circ T = id_E$.

2.1 Theorem

Let E, F be two families of finite dimensional Banach spaces such that E is c-chained to F. If X is a Banach space λ-locally E-factorable then X is λc-locally F-factorable.

Proof:

Let A be a finite dimensional subspace of X, and consider a locally factorable diagram for A with respect to a companion $E_A \in E$



By hypothesis, there is $G_A \in F$ and bounded linear maps $E_A \xrightarrow{T} G_A \xrightarrow{Q} E_A$ such that $Q \circ T = id_{E_A}$ with $\|T\| \leq c$, $\|Q\| \leq c$.

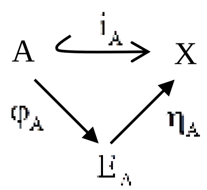
Thus the composition bounded linear maps $\psi_A = T \circ \varphi_A : A \to G_A$ and $\beta_A = \eta_A \circ Q : G_A \to X$ satisfy $\beta_A \circ \psi_A = i_A$ with $\|\psi_A\| \leq c\lambda$, $\|\beta_A\| \leq c\lambda$. That is, the maps ψ_A, β_A are bounded linear factorization of i_A through $G_A \in F$. Therefore X is λc-locally F-factorable. □

2.2 Theorem

Let X be a locally E-factorable Banach space, and let Y be a locally complemented subspace of X. Then Y is locally E-factorable.

Proof:

Let A be a finite dimensional subspace of Y, and consider a locally factorable diagram for A with respect to a companion $E_A \in E$



Since Y is a locally complemented in X, and $\eta_A(E_A)$ is a finite dimensional subspace of X, there is a bounded operator $p_A : \eta_A(E_A) \to Y$ such that p_A is the identity on $\eta_A(E_A) \cap Y$. Thus p(A) is the identity on A, since

$$A = i_A(A) = \eta_A(\varphi_A(A)) \subset \eta_A(E_A) \cap Y.$$

Therefore, $i_A = p_A \circ i_A = \eta_A \circ \varphi_A$. Hence Y is locally E-factorable. ∎

Since any Banach space X is locally complemented in its bidual space X^{**}, we have

2.3 Corollary

Any Banach space X has the same local factorable structure as X^{**}.

The following corollary is immediate since L_∞ spaces are locally complemented in any superspace.

2.4 Corollary

All L_∞ spaces are locally E-factorable whenever an L_∞ space is contained in a locally E-factorable space.

2.5 Corollary

All L_1 spaces are locally E^*-factorable whenever an L_∞ space is contained in a locally E-factorable space.

Proof:

It is easy to see that the family $\{\ell_\infty^n\}_{n=1}^{\infty}$ is chained to the family E.

Hence the family $\{\ell_1^n\}_{n=1}^{\infty}$ is chained to the family E^*. ∎

2.6 Example

The Schreier Space S is the completion of the space of finite sequences with respect to the following norm:

$$\|x\| = \sup_A \left(\sum_{j\in A} |x_j| \right),$$

where the supremum is taken over all admissible subset of N which are defined as the finite subsets $A = \{n_1, n_2, \ldots, n_k\}$ of N such that $n_1 < n_2 < \ldots < n_k$ and $k \le n_1$.

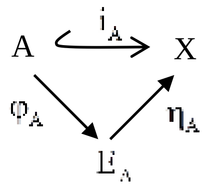
So, if S_k denotes the subspace of the Schreier space S generated by the first k elements of the canonical basis $\{e_i\}_{i=1}^{\infty}$, then every L_∞ space is locally $\{S_k\}_{k=1}^{\infty}$–factorable, since S contains isometric copies of the L_∞ space c_0.

2.7 Theorem

Let X be a Banach space which is λ-locally E-factorable and complemented in its bidual. Let U be an ultrafilter refining the order filter on the net C_X of the finite dimensional subspaces A of X. Then X is isomorphic to a complemented subspace of the ultraproduct $\left(\prod_{A\in C_X} E_A \right)_U$ of all companions $E_A \in E$ of $A \in C_X$.

Proof:

For each $A \in C_X$, consider a locally factorable diagram with respect to a companion $E_A \in E$



Let $(y_A)_U \in \left(\prod_{A\in C_X} E_A\right)_U$.

Then $\left(\|y_A\|_\infty\right)_{A\in C_X}$ is a bounded net, since

$$(y_A)_U = \left((y_A)+N_U\right) \in \ell_\infty(E_A;C_X)/N_U,$$

where $N_U = \left\{(y_A)_{A\in C_X} \in \ell_\infty(E_A;C_X) : \lim_U \|y_A\| = 0\right\}$.

Therefore, $(\eta_A(y_A))_{A\in C_X}$ is $\sigma\left(X^{**},X^{*}\right)$-bounded in X^{**}, since it is bounded in

$$X \hookrightarrow X^{**}.$$

Thus, the $\sigma\left(X^{**},X^{*}\right)$-closure of $\{\eta_A(y_A) : A \in C_X\}$ in X^{**} is compact.

Hence the weak-* limit of $(\eta_A(y_A))_{A\in C_X}$ over U exists.

Define a mapping $\Psi : \left(\prod_{A\in C_X} E_A\right)_U \longrightarrow X^{**}$ by

$$\Psi\left((y_A)_U\right) = \text{weak}^*\lim_U\left(\eta_A(y_A)\right).$$

If π is a projection of X^{**} onto X, and J_X is the natural isometric embedding of X onto $\left(\prod_{A\in C_X} A\right)_U$, then the composition maps

$$X \xhookrightarrow{J_X} \left(\prod_{A\in C_X} A\right)_U \xrightarrow{\Phi} \left(\prod_{A\in C_X} E_A\right)_U \xrightarrow{\Psi} X^{**} \xrightarrow{\pi} X$$

is the identity map on X, where $\Phi : \left(\prod_{A\in C_X} A\right)_U \to \left(\prod_{A\in C_X} E_A\right)_U$ is the map given by

$\Phi\left((x_A)_U\right) = \left(\varphi_A(x_A)\right)_U$.

That is $(\pi \circ \Psi) \circ (\Phi \circ J_X) = id_X$.

Therefore X is isomorphic to a complemented subspace of the ultraproduct $\left(\prod_{A\in C_X} E_A\right)_U$ of all companions $E_A \in E$ of $A \in C_X$. ∎

Using the proof of Theorem 3 with $E = \left\{\ell_p^n\right\}_{n=1}^{\infty}$, the following corollary is obvious.

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2.8 Corollary

If a Banach space X is locally $\{\ell_p^n\}_{n=1}^{\infty}$-factorable, $1 \le p \le \infty$, then X is an L_p space (or an L_2 if $1 < p < \infty$).

2.9 Example

(i) Consider the James space J, that is, the Banach space $(J, \|\cdot\|)$ of all real sequences $x = (a_1, a_2, \ldots)$ such that $\lim_{n\to\infty} a_n = 0$ and $\left(\sum_{i=1}^{n}\left(a_{p_{2i-1}} - a_{p_{2i}}\right)^2\right) < \infty$, where the supremum is taken over all choices of n and positive integers $p_1 < p_2 < \cdots < p_{2n}$, equipped with the norm

$$\|x\| = \sup\left(\sum_{i=1}^{n}\left(a_{p_{2i-1}} - a_{p_{2i}}\right)^2\right)^{1/2}.$$

The unit vector $\{e_n\}_{n=1}^{\infty}$ form a basis of J.

For each n, let $J_n = \operatorname{span}\{e_1, \ldots, e_n\}$, then J is $(1+\epsilon)$-locally $\{J_n\}_{n=1}^{\infty}$, and hence, is locally $\{J_n\}_{n=1}^{\infty}$–factorable.

(ii) Every separable Hilbert space is locally $\{J_n\}_{n=1}^{\infty}$-factorable.

Indeed, ℓ_2 is isomorphic to a complemented subspace of J and J is $(1+\varepsilon)$-locally $\{J_n\}_{n=1}^{\infty}$.

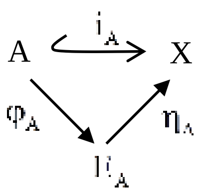
2.10 Theorem

Let E be a family of finite dimensional Banach spaces, and let Y be a Banach space complemented in its bidual. If $\operatorname{Ext}(W, Y) = 0$ for some Banach space W containing E uniformly complemented, then $\operatorname{Ext}(X, Y) = 0$ foe any Banach space X which is locally E-factorable.

Proof:

Let X be a Banach space which is λ-locally E-factorable, and let U be an ultrafilter refining the ordered filter on the net C_X of the finite dimensional subspaces A of X.

Let $F : X \to Y$ be a zero linear map and consider a locally factorable diagram for $A \in C_X$ with respect to $E_A \in E$



Then $F \circ \eta_A : E_A \to Y$ is a z-linear map.

So there is a constant t, independent of A and a linear map $L_A : E_A \to Y$ such that

$$\|F \circ \eta_A(y) - L_A(y)\| \le tZ(F \circ \eta_A)\|y\|, \ \ y \in E_A.$$

If $x \in X$, then $x = x_A \in A$ for some $A \in C_X$.

$$\begin{aligned}\|L_A \circ \varphi_A(x_A)\| &\le \|L_A\varphi_A(x_A) - F(\eta_A(\varphi_A(x_A)))\| + \|F(\eta_A(\varphi_A(x_A)))\| \\ &\le tZ(F\circ\eta_A)\|\varphi_A(x_A)\| + \|F(x_A)\| \\ &\le tZ(F)\|\varphi_A\|\|\eta_A\|\|x_A\| + \|F(x_A)\| \\ &\le t\lambda^2 Z(F)\|x_A\| + \|F(x_A)\|.\end{aligned}$$

Let $\Phi : \left(\prod_{A\in C_X} A\right)_U \longrightarrow \left(\prod_{A\in C_X} L_A(E_A)\right)_U$ be the map given by

$$\Phi((x_A)_U) = (L_A \circ \varphi_A(x_A))_U .$$

As in the proof of Theorem 3, the $\sigma\left(Y^{**}, Y^{*}\right)$-limit of $(L_A \circ \varphi_A(x_A))_{A\in C_X}$ over U exists in Y^{**}.

Hence, we can define a map $\Psi : \left(\prod_{A\in C_X} L_A(E_A)\right)_U \longrightarrow Y^{**}$ by

$$\Psi\left((L_A \circ \varphi_A(x_A))_U\right) = \text{weak}*\lim_U \left(L_A \circ \varphi_A(x_A)\right).$$

Given a projection π of Y^{**} onto Y and putting $L = \Psi\circ\Phi\circ J_X$ where J_X is the natural isometric embedding of X into $\left(\prod_{A\in C_X} A\right)_U$, we have for every $x \in X$,

$$\begin{aligned}\|F(x) - \pi L(x)\| &\le \|\pi\|\|F(x) - L(x)\| \\ &= \|\pi\|\|F(x) - \Psi\circ\Phi(x_A)_U\| \\ &= \|\pi\|\left\|\text{weak}*\lim_U (F(x) - L_A\circ\varphi_A(x_A)\right\| \\ &= \|\pi\|\left\|\text{weak}*\lim_U (F\circ\eta_A(\varphi_A(x_A)) - L_A(\varphi_A(x_A))\right\| \\ &\le t\lambda^2 Z(F)\|\pi\|\|x\|.\end{aligned}$$

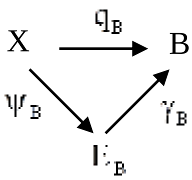
Thus F is trivial.

Therefore $\text{Ext}(X,Y) = 0$ for any Banach space X which is locally E-factorable.

□

2.11 Definition

Let E be a family of finite dimensional Banach spaces. A Banach space X is said to be **λ-colocally E-factorable** (or simply **colocally factorable** E) if there is a constant λ such that for every finite dimensional quotient B of X, there is $E_B \in E$, **a companion of** B, such that the quotient map $q_B : X \to B$ factors to bounded linear maps $\psi_B : X \to E_B$ and $\gamma_B : E_B \to B$ with $\|\psi_B\| \le \lambda$ and $\|\gamma_B\| \le \lambda$ through E_B, the diagram



is called **a colocally factorable diagram** for B.

2.12 Theorem

A Banach space X is λ-colocally E-factorable if and only if X^* is λ-colocally E^*-factorable.

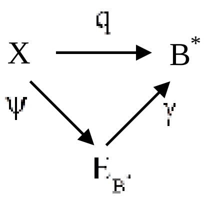
Proof:

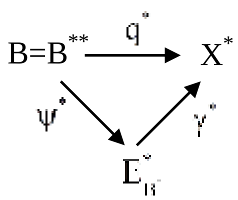
Suppose that X is λ-colocally E-factorable.

Let B be a finite dimensional subspace of X^*.

Since B is ω^*-closed, $B = (X/A)^*$ for some closed subspace A of X which implies that B^* is a quotient of X, since $B^* = (X/A)^{**} = X/A$.

Consider a colocally factorable diagram for B^*, and then take adjoints of its maps





we have a colocally factorable diagram for B, since

$\psi^* \circ \gamma^* = q^* = i_B$, $\|\gamma^*\| = \|\gamma\| \le \lambda$ and $\|\psi^*\| = \|\psi\| \le \lambda$.

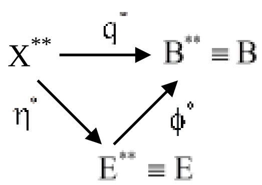
Therefore X^* is λ-locally E^*-factorable.

Conversely, suppose that X^* is λ-locally E^*-factorable.

Let $B = X/A$ be a finite dimensional quotient of X. Then $B^* = (X/A)^* = A^{\perp}$ is a subspace of X^* and the adjoint of the quotient map $q: X \to B$ is the inclusion map $q^*: B^* \to X^*$ of B^* into X^*, where $A^{\perp}$ is the annihilator of A.

Hence, there is $E^* \in E^*$ and bounded linear map $B^* \xrightarrow{\varphi} E^* \xrightarrow{\eta} X^*$ such that $\eta \circ \varphi = q^*$, with $\|\varphi\| \le \lambda$ and $\|\eta\| \le \lambda$.

If we take adjoints of the maps, we have



It is to see that $\phi^* \circ \eta^*\big|_X = q^{**}\big|_X = q$.

Therefore X is colocally E-factorable. □

2.13 Corollary

(i) All L_∞ spaces are colocally E-factorable whenever an L_∞ space is contained in a locally E-factorable space.

(ii) All L_1 spaces are colocally E^*-factorable whenever an L_∞ space is contained in a locally E-factorable space.

2.14 Theorem

If the dual X^* of a Banach space X is λ-colocally E^*-factorable, then X is $\lambda(1+\epsilon)$-locally E-factorable for every $\epsilon > 0$.

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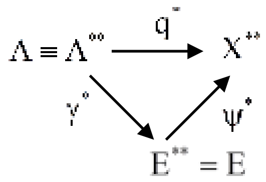
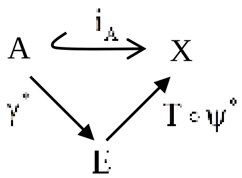
Proof:

Suppose that X^* is λ-colocally E^*-factorable.
Let A be a finite dimensional subspace of X.
Then $A^* = X/A^{\perp}$ is a quotient of X^*.
Hence, there is $E^* \in \mathbf{E}^*$ and bounded linear maps $X^* \xrightarrow{\psi} E^* \xrightarrow{\gamma} A^*$ such that $\gamma \circ \psi = q$ with $\|\psi\| \le \lambda$ and $\|\gamma\| \le \lambda$ where $q : X^* \to A^*$ is the quotient map.
If we take adjoints of the maps, we have



Since q^* is the inclusion map of $A \equiv A^{**}$ into X^{**}, we have
$A = q^*(A) = \psi^*(\gamma^*(A)) \subset \psi^*(E)$.
By the principle of local reflexivity, for every $\epsilon > 0$, there is an $(1+\epsilon)$-isomorphism $T : \psi^*(E) \to X$ such that $Tx = x$ for all $x \in \psi^*(E) \cap X$ which implies that
$(T \circ \psi^*) \circ \gamma^* = q^* = i_A$.
That is, we have the diagram



is a locally factorable diagram for A.
Therefore X is $\lambda(1+\epsilon)$-locally $\mathbf{E}$-factorable, since $\|T \circ \psi^*\| \le \lambda(1+\epsilon)$. □

2.15 Corollary

Let X be a Banach space. If X^{**} is locally $\mathbf{E}$-factorable (resp. colocally $\mathbf{E}$-factorable), then X is locally $\mathbf{E}$-factorable (resp. colocally $\mathbf{E}$-factorable).

2.16 Theorem

Let X and Y be Banach spaces and let $\psi_1 : X \to Y$, $\psi_2 : Y \to X$ be bounded linear operators such that $\psi_1 \circ \psi_2 = id_Y$. If X is colocally $\mathbf{E}$-factorable, so is Y.

Proof:

Suppose that X is colocally $\mathbf{E}$-factorable.
Then X^* is locally $\mathbf{E}^*$-factorable by Theorem 2.12.
Let B be a finite dimensional subspace of Y^*.
Then $A = \psi_1^*(B)$ is a finite dimensional subspace of X^*.
Hence there is $E^* \in \mathbf{E}^*$ and bounded linear maps $A \xrightarrow{\varphi} E^* \xrightarrow{\eta} X^*$ such that $\eta \circ \varphi = i_A$.
Put $\varphi_B = \varphi \circ \psi_1^*$ and $\eta_B = \psi_2^* \circ \eta$ where $\psi_1^* : Y^* \to X^*$ and $\psi_2^* : X^* \to Y^*$ are the adjoint maps of ψ_1 and ψ_2, respectively.
Thus the composition map $\eta_B \circ \varphi_B$ is the identity operator id_{Y^*} on Y^*.

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Hence Y^* is locally E^*-factorable which implies that Y is colocally E-factorable by Theorem 2.12. □

2.17 Corollary

If X is colocally E-factorable, so is every complemented subspace of X, and every Banach space isomorphic to X.

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